IGCSE MATHEMATICS

ENDORSED BY CAMBRIDGE INTERNATIONAL EXAMINATIONS

NOTES FOR GUIDANCE

Ric Pimentel and Terry Wall

To accompany ***IGCSE Mathematics*** *Student's Book* ISBN 0 7195 7458 7

First published in 1997
by John Murray (Publishers) Ltd
338 Euston Road
London NW1 3BH

Illustrations by Mike Humphries

Typeset in 10/12pt Times
Printed in Great Britain by St Edmundsbury Press Ltd, Bury St Edmunds, Suffolk

A catalogue entry for this title is available from the British Library

ISBN 0-7195-7462-5

CONTENTS

INTRODUCTION

The IGCSE Mathematics textbook has been written in direct response to the requests of teachers of the UCLES syllabus. When travelling abroad or when conducting training for overseas teachers in Cambridge, the most often heard comment was that there was not a straightforward, no frills textbook which covered the IGCSE syllabus at the extended level. The textbook tries to address this problem.

We are aiming at that range of students expecting to obtain a final grade at IGCSE between A* and D. We are aware that this is a considerable ability range and could result in the production of a very large and expensive book. We have therefore taken certain decisions about the book's presentation and what it should and should not contain.

It appears from comments made to us that teachers do not require a self-instructional textbook. For this reason we have reduced the 'teaching' aspect of the book: each chapter contains a number of worked examples which we hope teachers will work through on the board and supplement with other examples of their own. In addition to this, core material has only been included if we felt it was necessary for students to have a grasp of it before they embarked on the extended work. These sections can therefore be regarded as preparation material.

We know that groups of students vary considerably in their ability. As we mentioned previously, this text aims to cover this large range, so how the text is supplemented is left to the individual teacher.

We have followed the IGCSE syllabus very closely and the order of chapters is almost the same as the syllabus order. However, we are not suggesting that our chapter order is the order in which topics should be taught; on the contrary, we have deliberately not tried to produce a course plan. This too is left to the choice of the teacher and will depend upon the ability and motivation of the students.

Some of the chapters, Trigonometry, Mensuration and Algebraic Representation in particular, are quite long. Teachers may wish to split these chapters into sections and not to work through the whole chapter in one go. For most students this would probably be a sound approach.

Each chapter has Student Assessments which can be used in a variety of ways: in class to review completed work; as homework; as class tests; or left for a time and then used for revision purposes. Once again, the choice remains with the teacher. As a guide, the following pages highlight some of the main features within each chapter and, where applicable, give some suggestions as to methods of delivery.

Note:

1 Diagrams which look like rectangles can be taken to be so.

2 Diagrams which look like two-dimensional representations of cubes and cuboids are not normally labelled as such.

■ Essential Revision ■

(i) *ORDERING*

Order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, >, <, \geq, \leq$.

- Students will be familiar with the concept of ordering and will probably have come across most of the symbols $=, \neq, >, <, \geq, \leq$ before. The idea is to keep the initial concept as simple as possible and then for the teacher to develop the added complexities as appropriate.
- Teachers may wish to introduce the terms 'fewer than' and 'at least' for equal or greater although this will depend upon the student's grasp of vocabulary (see 'Linear Programming').
- This may be a point to discuss the difference between real numbers and integers.
- Teachers may wish to explain the meaning of the four Essential Revision sections at the beginning of this chapter.
- It should be discussed, particularly with brighter students, how discrete and continuous solutions are represented differently on a number line. Getting students to give examples of continuous and discrete data would be an integral part of this process.
- The use of calculators when writing a set of fractions in order of magnitude is left to the teacher's discretion.
- It is worth remembering that this is core level work and therefore examination questions will probably reflect this fact.

(ii) *STANDARD FORM*

Use the standard form $A \times 10^n$ where n is a positive or negative integer, and $1 \leq A < 10$.

- Many students may already be familiar with standard index form from their science classes. Discussion of astronomy and of molecular biology therefore makes a useful introduction to this chapter. Ideally teachers should aim to coordinate the teaching of this chapter with the use of standard form in science classes.
- We have stated that n is a positive or negative integer and have not mentioned, for the purposes of simplicity, that it can also be zero. This can perhaps be left to when the student is studying the zero index in the chapter on indices.
- It should also be emphasised that standard form makes comparison and ordering much simpler, and this is one of its major uses.

(iii) THE FOUR RULES

Use the four rules for calculations with whole numbers, decimal fractions and vulgar (and mixed) fractions, including correct ordering of operations and use of brackets.

- Most of the concepts dealt with in this chapter should be familiar to the majority of students. Teachers may wish to emphasise the order of operations to a greater extent than we have done here and to use some of their own examples. This is particularly valuable for weaker students who rely heavily on calculators and who are often surprised to be told that the answer on the calculator is wrong.
- For more able students this may be a useful time to discuss the use of the = sign, which is an equivalence relation but is becoming a short form of 'is equal to'. For example:

  the enlarged height is $8 \times \frac{3}{2}$ cm, i.e. 12 cm is now often written as
  the enlarged height is $8 \times \frac{3}{2}$ cm = 12 cm.
- Long multiplication and division are not specified in the IGCSE syllabus, but revision of these may provide a useful start.
- It is important for students to realise that questions such as $12 \div 3 \times 2$ can produce two answers, and that in such situations brackets should be used for clarity.
- In Exercise E some of the answers lead to recurring decimals. Teachers may wish to teach this unless students are already familiar with the concept.

(iv) ESTIMATION

Give approximations to specified numbers of significant figures (s.f.) and decimal places (d.p.) and round off answers to reasonable accuracy in the context of a given problem.

- Our rounding policy is the common one of 5 or more being rounded up, so that 5.5 becomes 6 and 6.5 becomes 7.
- Throughout the text we have used d.p. as shorthand for decimal places and s.f. as shorthand for significant figures.
- Teachers will need to explain to students that there is no hard and fast rule regarding appropriate accuracy. One or two decimal places is usually sufficient unless a greater deal of accuracy is specifically asked for.
- Teachers will need to decide whether any supplementary examples are needed to prepare students for Exercise C.
- It should be emphasised that IGCSE uses numbers to 3 s.f. unless otherwise stated. Teachers may therefore wish to emphasise the use of significant figures.

1 NUMBER, SET NOTATION AND LANGUAGE

Core Section

Use rational and irrational numbers; continue a given number sequence; recognise patterns in sequences and relationships between different sequences; generalise to simple algebraic statements (including expressions to the *n*th term) relating to such sequences.

- Prior work to rational and irrational numbers would include: fractions, decimals (including recurring decimals), an acquaintance with π, simple squares and square roots.
- Previous knowledge of sequences would include familiarity with sequences of odd and even numbers, square numbers and also triangle numbers. Some knowledge of Pascal's triangle and the Fibonacci series would be helpful but is not assumed.
- An opportunity for investigations and number problems leading to sequences would help place sequence work within a practical context and hence be useful in preparing for this chapter. Teachers will need to be supportive of students struggling to find a rule. It is worth emphasising that initially, a rule in words is just as good as an algebraic one. Most students will in the first instance need help in translating from one form to the other.

Extended Section

Use language, notation and Venn diagrams to describe sets and represent relationships between sets.

- The focus of the chapter is more about familiarising students with the new vocabulary and ways of setting out information.
- Teachers should note that the term $\subset$, though in the syllabus, is not in common use. Instead $\subset$ and $\subseteq$ are often regarded as synonymous.
- As a way of consolidating the new terminology it may prove useful to save the Student Assessments to act as revision tests a month or so after the chapter has been studied.
- Teachers may wish to omit Questions 10 and 11 in Exercise 1.6 or to give students some extra help.
- Question 4 in Exercise 1.10 may need further discussion and a definition of $(P \cup C \cup B)$ may need to be given.

2 LIMITS OF ACCURACY

Core Section

Give appropriate upper and lower bounds for data to a specified accuracy (e.g. measured lengths).

- The concept of degrees of accuracy is quite difficult for some students. We have kept our explanation quite short, but suggest that teachers spend some time working through similar examples before students begin Exercise 2.1.
- Particular attention needs to be paid to the correct notation used on the number lines.
- Teachers will need to assist all but the most able students when upper and lower bounds are required for negative numbers, as in Exercise 2.1 Questions 1(f) and 2(f).

Extended Section

Obtain appropriate upper and lower bounds to solutions of simple problems (e.g. the calculation of the perimeter or the area of a rectangle) given data to a specified accuracy.

- If the original concept is clearly understood then the extension into a maximum and minimum product should not prove too problematic. Here too though we suggest that a number of similar questions should be worked through with the class before starting Exercise 2.2.
- We have introduced the term 'limits' as well as 'bounds' as students may have met this term in Physics.

3 RATIO, PROPORTION AND MEASURES OF RATE

Core Section

Demonstrate an understanding of the elementary ideas and notation of ratio, direct and inverse proportion and common measures of rate; divide a quantity in a given ratio.

- We have given examples of both the unitary and ratio methods. Both have their strengths and weaknesses. Less able students seem to find the unitary method easier to grasp.
- Exercise 3.4 Question 4(b) may give teachers the opportunity to discuss realistic answers (and questions), e.g. Would 48 men get in each other's way?

Extended Section

Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities; increase and decrease a quantity by a given ratio.

- One of the main aims for a teacher in this section is to get students to feel comfortable with the use of the symbol of proportionality and the constant *k*.
- Teachers may need to define origin and gradient.
- Some extra examples may be needed to reinforce Exercise 3.5 Questions 2 and 3.

4 PERCENTAGES

Core Section

Calculate a given percentage of a quantity; express one quantity as a percentage of another; calculate percentage increase or decrease.

- We have made no comment in the chapter about the use of calculators, but we do feel that students should know some of the equivalent fractions and decimals, for example, of 50%, 25%, $33\frac{1}{3}$%, 20% etc. and be able to use these in calculations without using a calculator.
- When working with either percentage increases or decreases, some teachers may wish to use the method of working out the increase or decrease and then adding or subtracting, respectively. We prefer the method of showing the original price as 100% and then using the appropriate multiplier.
- Some questions ask students to calculate the increase or decrease only. They must, as ever, be taught to read each question carefully.
- Students should be aware of problems involving change of units.

Extended Section

Carry out calculations involving reverse percentages, for example, finding the cost price when given the selling price and the percentage profit.

No additional notes.

5 GRAPHS IN PRACTICAL SITUATIONS

Core Section

Interpret and use graphs in practical situations including conversion graphs, distance–time graphs and travel graphs.

- Conversion graphs can prove very useful in emphasising the point that if a relationship exists between two variables then it can be plotted. Once plotted it can be used effectively to determine the value of one variable if the other is known.
- When a number of conversions are needed as in Exercise 5.1 Questions 2 and 3, these types of graph are efficient to use.
- More able students will probably point out that the results obtained from a conversion graph are not as accurate as those obtained by applying the rule governing the two variables. It will therefore be necessary to point out that conversion graphs are particularly useful when plotted from experimental results, which tend not to follow a precise rule. Undertaking practical work in science lessons will help emphasise this point.
- With distance–time travel graphs, an understanding that the gradient of the line determines the speed is essential. Whether students have truly grasped this is tested in Core Assessment 2 Question 4.

Extended Section

Apply the idea of rate of change to easy kinematics involving speed–time graphs, acceleration and deceleration; calculate distance travelled as area under a linear speed–time graph.

- In this Section we have used the terms acceleration and deceleration to represent positive and negative acceleration respectively. Teachers may wish to point out that deceleration can be written as a negative acceleration. For example, the worked example (iii) asks for the deceleration between 14 and 16 seconds. The answer is given as deceleration = $8\,\text{m/s}^2$, but this could have been given as acceleration = $-8\,\text{m/s}^2$.
- Teachers could take this opportunity to explain how the units m/s^2 are obtained from the graph.
- An understanding that a horizontal line on a speed–time graph does not imply that the object is stationary is important. Comparing this to the distance–time graphs is recommended.
- If access to motion sensors is possible, then their use would greatly improve students' understanding of these graphs.

6 *ALGEBRAIC REPRESENTATION AND MANIPULATION*

Core Section

Use letters to express generalised numbers and express basic arithmetic processes algebraically, substitute numbers for words and letters in formulae; transform simple formulae; use brackets and extract common factors.

- We have chosen to expand brackets before dealing with factorisation, and then to cover substitution; but the order of this section is the teacher's choice.
- From the worked example in 'Transformation of formulae', teachers will see that we prefer the more logical $a - 2b = \boldsymbol{c}$ rather than many students' preference for a further rearrangement to make $\boldsymbol{c} = a - 2b$. It is worth spending time explaining that the subject does *not* have to be on the left.

Extended Section

Expand products of algebraic expressions; factorise and simplify expressions; manipulate algebraic fractions; transform more complicated formulae.

- Our method of showing expansions using a grid is quite widespread; however, there are other methods which are equally effective.
- Our method of factorising quadratic expressions refers back to the grid introduced at the start of this Section. Teachers may choose a different approach here too.
- The note on worked example (d) in the section on 'Transformation of complex formulae' needs careful explanation and could perhaps be backed up with some numerical examples. However, it will need to be stressed that numerical examples do not constitute a proof.
- We feel that Exercises 6.15 and 6.16 progress in difficulty at a rate which will extend able students. For groups of average ability, teachers may need to add some supplementary questions.

7 EQUATIONS AND INEQUALITIES

Core Section

Solve simple linear equations in one unknown; solve simultaneous linear equations in two unknowns.

- In our examples we have not shown all the intermediate steps in the solving of equations, i.e.

  $3x + 8 = 14$
  $3x + 8 – 8 = 14 – 8$
  $3x = 6$
  $\frac{3x}{3} = \frac{6}{3}$
  $x = 2$

  Some teachers may prefer this fuller method, but our experience has suggested that students will be familiar with equations by this stage and can therefore solve them by showing only some of the steps. It is however important to insist that some working out is given, even if the student is capable of solving it in his/her head.
- Simultaneous equations can be solved by either of the two methods shown. In Exercise 7.2 some teachers may wish students to solve the equations by only one or by both methods. However, it is suggested that more able students should try to identify when one method is simpler to use than the other.

Extended Section

Construct equations from given situations; solve quadratic equations by factorisation and either by use of the formula or by completing the square; solve simple linear inequalities.

- In Exercise 7.5 it should be apparent to students that the diagrams are not meant to be to scale, but some students may need this to be emphasised.
- Some teachers may wish to leave the Section on ‘Solving quadratic equations’ until they work on graphs of quadratic functions. Certainly the strongest students will be able to appreciate a simple graphical illustration at the same time as learning how to solve the equations by factorising.
- Students of average ability may have considerable difficulty with the concept of linear inequalities. It may therefore be necessary to give them more worked examples than those provided in the text.

8 STRAIGHT LINE GRAPHS

Calculate the gradient of a straight line from the coordinates of two points on it; calculate the length of a straight line segment from the coordinates of its end points; interpret and obtain the equation of a straight line graph in the form $y = mx + c$; graphical solution of simultaneous equations.

- Students will rarely calculate the gradient of a straight line by using negatives, as shown in the note on the first worked example. But it is useful here to tell them that the order in the numerator and the denominator must be the same, i.e.

$$\frac{y_1 - y_2}{x_1 - x_2} \quad \text{not} \quad \frac{y_1 - y_2}{x_2 - x_1}$$

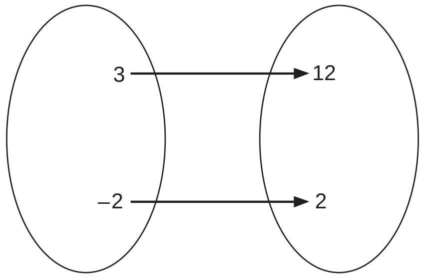
- In Exercise 8.1 we have deliberately put in 'if necessary' depending on the ability of the student. However, teachers may wish to insist that graphs are drawn out at least in the early questions, to prevent the ordering problems mentioned above.
- When calculating gradient and intercept we suggest that teachers put diagrams on the board to illustrate worked examples (a) and (b). It may also be necessary to work through at least one part of each of the questions in Exercise 8.5.
  [Note: Strictly speaking $\frac{3x - y}{y} = 2$ is not the same as $y = x$ because it does not contain the point (0,0). The origin is also excluded from other equations in this Exercise.]
- When plotting a line from its equation, teachers should stress to students that the line drawn is not restricted to lying between the two given points. Extending the line beyond the given points is therefore a good way of reinforcing this.
- How the graphical solution of simultaneous equations is linked to earlier work on simultaneous equations is left up to the teacher to decide. However, it is important that a link should be made, otherwise many students will consider them as two unrelated topics.

9 *FUNCTIONS*

Use function notation, e.g. $f(x) = 3x - 5$, $f: x \rightarrow 3x - 5$ to describe simple functions, and the notation $f^{-1}(x)$ to describe their inverses; form composite functions as defined by $gf(x) = g(f(x))$.

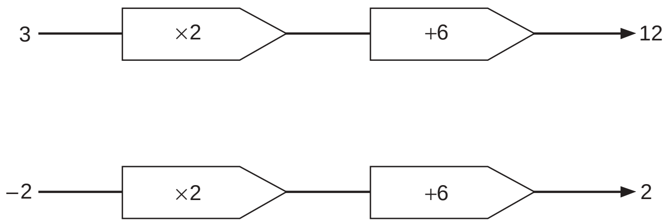
- The initial concepts are not too difficult in this chapter. It may be that students have not previously been introduced to function notation. Teachers may therefore wish to make up some preliminary exercises to make students familiar and comfortable with the use of function notation $f(x)$ and $f: x \rightarrow$ before starting the chapter.
- Since we have not produced a self-instructional text we have left teachers to decide how they will teach 'inverse functions'. Teachers may choose from the two approaches noted below to represent $f: x \rightarrow 2x + 6$.

  1 Use of diagrams:



  which allows us to talk of going 'backwards'.

  2 Use of function machines:



  and then reversing this process.

  Each of these approaches leads into the idea that not every function has a real inverse.
- Composite functions tend to be more problematic for students to grasp. In order to get the concept across, we recommend that lots of examples are done on the board first. We have used the notation $fg(x)$, but some teachers may be more familiar with $f(g(x))$.

10 *GRAPHS OF FUNCTIONS*

Core Section

Construct tables of values for functions of the form $\pm ax^2 + bx + c$, a/x ($x \neq 0$) where a, b and c are integral constants; draw and interpret such graphs; solve linear and quadratic equations approximately by graphical methods.

- Previous knowledge would include familiarity with straight line and curved graphs. Teachers may wish to revise earlier work on graphs and to make sure that students are familiar with the vocabulary of this chapter.
- We have given the simpler format $y = 2x^2 + 3x - 12$ as an example of a quadratic function. Some teachers will want to express this either as:

  $f(x) = 2x^2 + 3x - 12$ or
  $f{:}x \rightarrow 2x^2 + 3x - 12$.

- In the worked example we prefer the use of the term 'domain' rather than 'range'. This is a subtle point which some teachers may prefer to ignore.
- Exercise 10.1 may need to be supplemented by further questions. We have deliberately chosen questions where the solutions can in fact be read off from the tables of values. We have found that initially students are happier practising drawing graphs where this is the case, rather than where the solution calls for an estimate.

Extended Section

Construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant and $n = -2, -1, 0, 1, 2, 3$ and simple sums of not more than three of these and for functions of the form a^x where a is a positive integer; estimate gradients of curves by drawing tangents; solve associated equations approximately by graphical methods.

- We have chosen the term reciprocal function for $f(x) = \frac{1}{x}$ rather than the inverse function.
- We have assumed that students will be familiar with the term gradient and will be aware of how to draw tangents to a curve.
- From our experience, the Section on 'Solving equations by graphical methods' needs to be accompanied by considerable teacher input. Our worked examples should therefore only be seen as a framework from which the teacher will need to build a series of lessons.
- It is particularly important that students are made aware of the general shape of a reciprocal function. Questions which require students to plot integer values for the function need to be flagged-up as needing care, as the shape of the graph in the domain $-1 < x < 1$ will not always be obvious.
- We have made no reference to the use of graphical calculators or computers, for two reasons. Firstly, the examination regulations governing the use of different models of graphical calculators seem to change quite often. Secondly, individual countries seem to have adopted varying positions with respect to the use of computers.

Note: In examinations, candidates must abide by UCLES regulations.

11 INDICES

Core section

Use and interpret positive, negative and zero indices.

- Students will certainly have come across squares and cubes, square roots and probably cube roots too.
- Teachers may wish to show that $a^2 \times a^3 = a^5$ by $(a \times a) \times (a \times a \times a)$ and treat division of powers and raising a power to a further power in a similar way.
- With negative indices, teachers may wish to illustrate $a^{-m} = \frac{1}{a^m}$ either as we have or to start with $\frac{1}{a^m}$ and show this to become a^{-m}.
- Some students may require additional help with Exercise 11.4 Question 3 and Exercise 11.5 Questions 2, 3 and 4, but we suggest that they be encouraged to try to solve the problems themselves first.

Extended section

Use and interpret fractional indices.

- In Student Assessment 1 Question 3, all students should be familiar with the idea of drawing a pair of axes. Teachers may therefore wish to rephrase this for the most able students as:

  Draw out a graph for $y = 3^{x/2}$ for $x \leftarrow \{-4, -3, \ldots, 3, 4\}$ and $y \leftarrow \{0, 1, 2, \ldots, 9, 10\}$

12 LINEAR PROGRAMMING

Represent inequalities graphically and use this representation in the solution of simple linear programming problems (the conventions of using broken lines for strict inequalities and shading unwanted regions will be expected).

- In our revision we have restricted the meaning of $\geq$ to 'is greater than or equal to'. Teachers may wish to point out that this also means 'is not less than'.
- The terms 'fewer than' and 'at least' may have been discussed in the chapter on 'Ordering'. Teachers may need to give help with this chapter depending on the level of vocabulary of the students.
- Students will need to be made aware of the different conventions used in shading. They may be asked either to shade the region satisfied by the inequalities, or to leave it unshaded. If students are asked simply to identify the region which satisfies the given inequalities, then we suggest that the clearest way of doing this is to leave it unshaded.
- At some point the teacher will probably introduce the term 'feasible solution'. We suggest that this is best done before Exercise 12.4.

Note: In IGCSE examinations the questions will only require shading the unwanted region.

■ Essential Revision ■

LOCI

Use the following loci and the method of intersecting loci for sets of points in two dimensions:

- which are at a given distance from a given point,
- which are at a given distance from a given straight line,
- which are equidistant from two given points,
- which are equidistant from two given intersecting straight lines.

- Although this chapter is at core level we have tried to include questions which would challenge the more able students too.
- The term locus and the idea of the path of a moving object are considered to be the same. (At this level we feel that this is a readily understood concept.) Some teachers may wish to discuss differences between 'a locus' and 'a path' with more able students.
- For Exercise A Question 6, teachers may wish the student to sketch the coastline. They should also be able to choose a suitable scale.
- Exercise A Question 8 will also help students with the concept of Symmetry, covered fully in Chapter 19.
- Exercise C Questions 4 and 5 are outside the syllabus, but we feel are interesting questions.
- Student Assessment 1 Questions 4 and 5, and Question 5 in Student Assessment 2 may seem quite difficult in that similar problems have not been dealt with in the text. However, their purpose is to encourage thought. After the assessment they will also lead to some valuable discussion between student and teacher.

13 GEOMETRICAL RELATIONSHIPS

Use the relationships between the areas of similar triangles, with corresponding results for similar figures and extension to volumes of similar solids.

- Teachers may wish to emphasise the difference between the word 'similar' used in everyday speech and its particular mathematical definition (possibly linking it to the use of the word 'average' dealt with in Chapter 21 on Handling Data).
- This may be the point to talk about enlargement, reduction and scale drawings.
- Some teachers may wish to prove why the ratio of the areas of similar triangles is k^2, though this will depend largely on the ability of the group.
- We have found that students find work on scale factors of enlargement and reduction difficult; it may therefore be necessary to reinforce the concepts with extra examples.
- In Exercise 13.1 students may find Questions 4 and 5 easier if they redraw the diagram for the questions as two separate triangles and then mark on all known measurements.
- Exercise 13.4 may prove difficult for some students. In particular for weaker students it may need to be pointed out that in Questions 4 and 5 the shapes in question are not mathematically similar.

14 *ANGLE PROPERTIES*

Core Section

Calculate unknown angles using the following geometrical properties:

- angle properties of regular polygons,
- angle in semi-circle,
- angle between tangent and radius of a circle.

No additional notes

Extended Section

Use the following geometrical properties:

- angle properties of irregular polygons,
- angle at the centre of a circle is twice the angle at the circumference,
- angles in the same segment are equal,
- angles in opposite segments are supplementary.

No additional notes

15 *TRIGONOMETRY*

This is one of the biggest chapters in the book and some of the later questions can be quite difficult for many students. We suggest that before this chapter is attempted for the first time, the teacher plans its delivery thoroughly. Questions the teacher may wish to consider are:

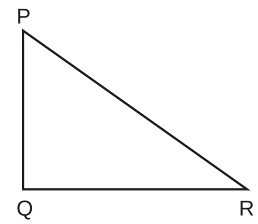
- how many sections to break the topic into,
- how to transfer students from work on right-angled triangles to non-right-angled triangles and what part the sine and cosine curves will play in this,
- whether to relate the sine and cosine rule to right-angled triangles,
- how to ensure adequate revision of this topic.

This chapter may well be tackled by firstly undertaking departmental in-service training.

Core Section

Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles, to the calculation of a side or of an angle of a right-angled triangle.

- Teachers may wish to introduce students to the convention whereby point A can also be labelled angle A. In the same way a, b, c would refer to the sides opposite to the points A, B and C.
- In the triangle PQR below, teachers may wish to use PR or q in trigonometric formulae.



  We would suggest that since students are likely to come across both types, it may be useful to use both when going through worked examples.
- In the worked example (b) of 'Pythagoras' Theorem', we have chosen to rearrange the formula first and then to substitute in the values. Teachers should decide for themselves whether they prefer to follow this method or to substitute first. Similarly in Exercise 15.4 Question 2, it will depend on the ability of the students whether and when they stop bothering to square root intermediate answers. Teachers may wish to show the limitations of calculators in this type of situation if students square root, round off and then square again.
- In Exercise 15.4 Question 5, teachers will want to draw the distinction between Pythagoras and its converse.
- From Exercise 15.4 Question 3 onwards we have used one letter to represent both an object and its position for the sake of simplicity.

Extended Section

Solve trigonometrical problems in two dimensions involving angles of elevation and depression; extend the sine and cosine functions to angles between 90° and 360°; solve problems using the sine and cosine rules for any triangle and the formula area of triangle $= \frac{1}{2}ab \sin C$; solve simple trigonometrical problems in three dimensions including angle between a line and a plane.

- A little practical work on angles of elevation and depression is very useful here, especially if students have access to a clinometer and can work outside.
- We have deliberately not put in the teachers' work which extends the probable assumption by many students that the trigonometric functions are only applicable for acute angles. Once again it is important that a department discusses the method by which it intends to tackle this concept.
- In Exercise 15.9 Question 2 the teacher may choose to spend time on the ambiguous case with all students, or see if some students raise the point themselves.
- Some teachers may also wish to investigate with students what happens if either the sine or cosine rule is applied to right-angled triangles.
- As we stated earlier, when referring to Pythagoras' theorem, we have rearranged formulae and then substituted. In using the cosine rule, we have used this method and also the other one, where the substitution takes place first. As with much of this chapter, the way it is taught will depend upon the level of ability of the students and the teachers' own preferences.

16 *MENSURATION*

Core Section

Carry out calculations involving the circumference and area of a circle, the area of a parallelogram and a trapezium, the volume of a prism and cylinder, and the surface area of a cuboid and a cylinder.

- We have rearranged circle formulae before substituting in numerical values. Some teachers and students may wish to substitute the numbers first and then rearrange the formulae.
- At core level, it is recommended that students draw diagrams to assist with visualisation of the problem in Exercise 16.7.

Extended Section

Solve problems involving the arc length and sector area as fractions of the circumference and area of a circle, the surface area and volume of a sphere, pyramid and cone (given the formulae for the sphere, pyramid and cone).

- In the worked examples on the area of a sector we have departed from our usual practice and substituted before rearranging to find 'r'. We think that all but the most able will find this easier to understand.
- Many of the later questions in Exercises 16.12, 16.14, 16.17 and 16.21 are difficult. Although students will have covered the necessary theory by this stage, the questions are aimed at stretching the more able.
- In Exercises 16.16 to 16.20 we have usually referred to shapes as pyramids and cones rather than right pyramids and right cones. However, in each case the apex is vertically above the centre of the base, unless otherwise stated.

17 *VECTORS*

Core Section

Describe a translation by using a column vector, $\overrightarrow{AB}$ or **a**; add vectors and multiply a vector by a scalar.

- This chapter has been covered at a theoretical level. Teachers wishing to introduce the topic in a more practical way can incorporate games such as ‘vector golf’ or use ‘Treasure Island maps’ whereby the route to the treasure is given as a succession of vectors.
- Teachers may wish to point out that vectors are examples of a column matrix, more of which is discussed in Chapter 18.

Extended Section

Calculate the magnitude of a column vector. (Vectors will be printed as $\overrightarrow{AB}$ or **a** and their magnitudes denoted by modulus signs, e.g. $|\overrightarrow{AB}|$ or $|a|$. In their answers to questions, candidates are expected to indicate **a** in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{AB}$ or $\underline{a}$.)

Represent vectors by directed line segments; use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors; use position vectors.

- When using Pythagoras to calculate the magnitude of a vector, teachers may wish to point out that any negative sign can be ignored as in worked example (ii):

  $|\overrightarrow{BC}| = \sqrt{(-6)^2 + 8^2}$ can also be written as $|\overrightarrow{BC}| = \sqrt{6^2 + 8^2}$

  $= \sqrt{100}$ $\quad$ $= \sqrt{100}$

  $= 10$ $\quad$ $= 10$

- When dealing with the work on vector geometry, we have assumed that students are familiar with the geometry of plane shapes such as hexagons, parallelograms and isosceles triangles.

18 MATRICES

Display information in the form of a matrix of any order; calculate the sum and product (where appropriate) of two matrices; calculate the product of a matrix and a scalar quantity; use the algebra of 2×2 matrices including the zero and identity 2×2 matrices; calculate the determinant and inverse $\mathbf{A}^{-1}$ of a non-singular matrix $\mathbf{A}$.

- Matrices tends to be a topic that a lot of students find difficult to grasp. We therefore recommend that teachers cover the work slowly and give plenty of worked examples.
- Most of the matrices used throughout the chapter are of a hypothetical nature. However, students should not lose track of the fact that matrices are a way of storing information.
- Teachers may need to clarify to students what a scalar quantity is, especially if they have not come across the terminology in Physics.
- Although non-singular matrices do not form part of the syllabus, we thought that their inclusion was important in order for students to appreciate why it was that some matrices had no inverse, as in Exercise 18.8.
- The use of graphical calculators greatly speeds up the rate at which students can solve problems involving matrices. Once again, however, their use is left to the discretion of the individual teacher.

19 SYMMETRY

Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone); use the following symmetry properties of circles:

- equal chords are equidistant from the centre,
- the perpendicular bisector of a chord passes through the centre,
- tangents from an external point are equal in length.

- Teachers may wish to cover this chapter in two parts. The first part could be the symmetry properties of named solids, and the second the work involving circles.
- Teachers may wish to consider parallelograms here.
- The work on the symmetry properties of solids is best done in class with the use of solid models.
- When we talk of pyramids and cones, we have taken this to mean right pyramids and right cones unless otherwise stated. Exercise 19.2 can be used as a discussion exercise. Question 3 in particular can be extended to include a variety of permutations of the same basic idea.
- It will obviously be very useful if the formal proofs of congruency are at least touched upon, particularly with the more able students.
- The level of ‘proof’ required in Question 3 of both Student Assessments will vary depending on how much work has been done previously in class.

20 TRANSFORMATIONS

Core Section

Recognise and describe reflections, rotations, translations and enlargements.

- Although a long section, most of the concepts covered should already be familiar to most students.
- With any work on symmetry, a useful starting point would be to discuss where they have seen symmetrical shapes in the real world. Students will then appreciate that the quality which makes so many patterns appealing is their symmetry.
- A display of the use of symmetry in different cultures is also recommended.
- With reflections, the use of mirrors will help students pick up any errors they have made. (Errors are most likely to occur where the mirror line does not lie either horizontally or vertically; this is not necessary at core level.)
- Similarly, tracing paper could be used when checking work involving rotations.
- The idea that enlargement can also represent an object becoming proportionally smaller is sometimes confusing for students. The value of the scale factor of enlargement therefore determines whether the object gets 'bigger' or 'smaller'.

Extended Section

Use the following transformations of the plane: reflection, rotation, translation, enlargement, shear, stretching and their combinations.

Identify and give precise descriptions of transformations connecting given figures; describe transformations using coordinates and matrices (singular matrices are excluded).

- Students will need to have covered the chapter on straight line graphs before attempting the extended work on reflection.
- When students are asked to find the centre of a rotation by construction as in Exercise 20.12, it is essential that the need for accuracy is stressed. Students should check the accuracy of their diagrams by using tracing paper.
- Students will need to have covered the work on matrices before tackling Exercise 20.18 onwards.
- It is expected that all but the most able students will find the exercises on matrices and transformations difficult. It is therefore recommended that teachers spend time going through a number of examples beforehand on the board.

21 *HANDLING DATA*

Understand and use mean, median and mode. Construct and read histograms with equal and unequal intervals (areas proportional to frequencies and vertical axis labelled 'frequency density'); construct and use cumulative frequency diagrams; estimate the median, percentiles, quartiles and inter-quartile range; calculate an estimate of the mean for grouped and continuous data; identify the modal class from a grouped frequency distribution.

- A general discussion about the increasing use of statistical information to support an argument may be useful before starting this chapter.
- Students could be encouraged to make a display of graphs and other statistical information appearing in newspapers and magazines and in particular, to comment about the relative effectiveness of each of the types of graph used.
- Teachers should inform students that the word 'average' in particular is used to represent either mean, median or mode, depending on which is the most favourable in making a particular point.
- This topic, in particular, lends itself to plenty of practical work. Teachers are therefore encouraged to incorporate collecting some 'real' data into their lessons. By collecting 'real' data and applying statistical analysis to it, the students become more motivated and the work itself becomes both relevant and enjoyable. A combination therefore of text questions and similar questions applied to their own data is probably best.
- Exercise 21.7 Question 4 asks the students to write a short report. Students may need some guidance for this but we feel this to be a very useful skill. Teachers may wish to add this to Question 5 also.

22 PROBABILITY

Calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate (in possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of branches).

- Students will need to compare the theoretical aspects of probability with the practical reality of the results. An understanding that probability is purely the study of chance and that predictions may not actually happen is an important concept to get across.
- A discussion about equally likely outcomes (with a fair dice) and sporting outcomes, where the outcomes are not equally likely, would be a useful introduction to the topic.
- Teachers often assume that students are familiar with a pack of playing cards. This is not always the case, and therefore knowledge of this will be necessary before many of the questions can be tackled.
- We have tried to incorporate both simple and more complex tree diagrams.
- Students will need to appreciate that the wording ‘and’ and ‘or’ gives a clue as to whether probabilities are added or multiplied together.
- The Assessments at the end of the chapter are quite long. Teachers may therefore have to select which questions students attempt, or allow more time for the Assessment’s completion.

22 PROBABILITY

Calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate (in possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of branches).

- Students will need to compare the theoretical aspects of probability with the practical reality of the results. An understanding that probability is purely the study of chance and that predictions may not actually happen is an important concept to get across.
- A discussion about equally likely outcomes (with a fair dice) and sporting outcomes, where the outcomes are not equally likely, would be a useful introduction to the topic.
- Teachers often assume that students are familiar with a pack of playing cards. This is not always the case, and therefore knowledge of this will be necessary before many of the questions can be tackled.
- We have tried to incorporate both simple and more complex tree diagrams.
- Students will need to appreciate that the wording ‘and’ and ‘or’ gives a clue as to whether probabilities are added or multiplied together.
- The Assessments at the end of the chapter are quite long. Teachers may therefore have to select which questions students attempt, or allow more time for the Assessments’ completion.